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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

ENGINEERING METHODS OF CALCULATION OF DIFFERENT REGIMES OF HEATING OF THERMALLY MASSIVE OBJECTS IN METALLURGICAL HEAT TECHNOLOGIES UNDER COUNTERCURRENT CONDITIONS. 1. STATE OF THE PROBLEM. CONVECTIVE HEATING

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Methods of study of countercurrent heat exchange have been analyzed, and an approximate procedure of mathematical modeling of convective heat exchange in metallurgy has been given.

Introduction. Heat exchange of parts of a solid body and a gas, moving in opposition, is widespread in industry and primarily in metallurgy. Cases of such heat exchange include heating of a charge in blast furnaces and of metal billets in continuous furnaces, cooling of pellets in stack-type coolers, dry quenching of coke, etc. Countercurrent processes of heat transfer are studied with different degrees of accuracy in [1–5].

Engineering computational procedures are most frequently based on the following simplifying assumptions [2]:

(1) particles in the bed are considered as thermally thin insulated bodies;

(2) heat exchange in the bed follows the Newton law (convection);

(3) heat loss to the ambient medium is negligibly small;

(4) there are no internal heat sources and sinks in the material heated;

(5) heat transfer by conduction is disregarded.

All five hypotheses are quite justified in countercurrent heat exchange in a dense bed of finely divided materials. In all other cases they can cause substantial errors, in particular, in thermal calculations of heating of a metal in continuous furnaces.

Formulation of the Problem. Early progress in theoretical investigation of countercurrent heat exchange is associated with [6, 7] (these works were devoted to calculation of the corresponding processes of production of cast iron). A blast furnace is a typical countercurrent apparatus in which high efficiency of heat exchange is ensured precisely by the countermotion of a gas and a charge.

It has been proposed [6] that the regularities of countercurrent heat exchange between the material and the gas be found by solution of the following problem (see Fig. 1).

A bed of height H, consisting of pieces of the same size and shape, descends with a constant rate w in a stack of constant cross section with area S. The pieces of the material with temperature T'_m (the same throughout the volume) that are charged at the top are heated in the process of descent and have temperature T''_m at exit from the heat-exchange zone. The initial temperature of the gas blown from the bottom upward is equal to T'_g and the temperature of the gas at exit from the bed is equal to T''_g. The volumetric rate of flow of the gas V_g is constant. The heat capacities of the material c_m and the gas c_g remain constant in the process of heat exchange. The other thermophysical characteristics are also constant.

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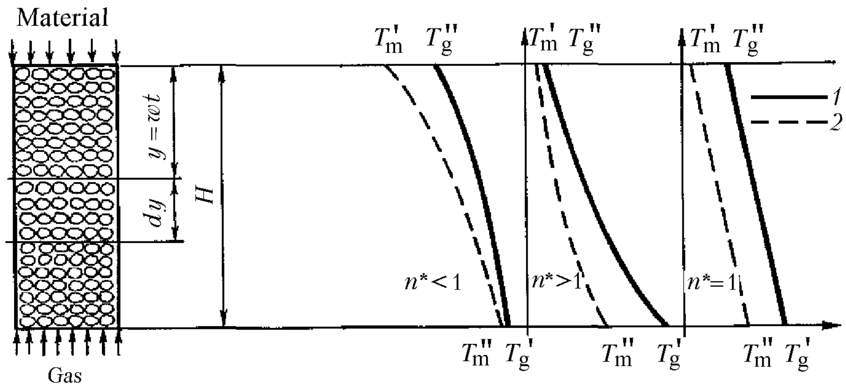

Fig. 1. Scheme of countercurrent heat exchange in the bed: 1) gas; 2) material.

With allowance for what has been said above, we consider the so-called "quasistationary" process in which the temperature on any horizon $y = wt$ of the bed remains constant with time but changes from horizon to horizon. It is necessary to find the distribution of the temperature fields of the material and the gas over the bed's height.

Depending on the value of the parameter n^* (ratio of the heat capacities or water equivalents of the material W_m and the gas W_g, $n^* = W_m/W_g$), we recognize three fundamentally different cases of countercurrent heat exchange: $n^* < 1$, $n^* > 1$, and $n^* = 1$.

When $n^* < 1$ the heat exchange is described by the equation

$$G_m c_m dT_m = \alpha_V (T_g - T_m)\, S dy\,. \tag{1}$$

Here $G_m = W_m \gamma_m S(1 - \Pi)$ is the mass flow rate. Equation (1) characterizes a change in the enthalpy of the material, which is numerically equal to the quantity of heat received by the bed from the gas. For this case the difference of the enthalpies of the gas and charge flows on any horizon y is equal to the difference of the enthalpies of the gas at exit and of the charge at entry, i.e., $W_g T_g - W_m T_m = W_g T''_g - W_m T'_m$. With the simplification $T'_m = 0$ we obtain

$$T_g(y) = T''_g + n^* \bar{T}_m(y)\,. \tag{2}$$

Substituting (2) into (1), we arrive at the ordinary differential equation

$$\frac{d\bar{T}_m}{dt} + \frac{\alpha_V S\,(1 - n^*)}{W_m}\bar{T}_m = \frac{\alpha_V S}{W_m} T''_g\,. \tag{3}$$

The solution of Eq. (3) with the boundary condition $\bar{T}_m(y)\big|_{y=0} = T'_m = 0$ has the form $\bar{T}_m(y) = \dfrac{T''_g}{1-n^*}\left\{1 - \exp\left[-\dfrac{\alpha_V S}{W_m}(1-n^*)y\right]\right\}$. If we take into account that we must have $\bar{T}_m = T'_g$ (completed heat exchange) when $y \to \infty$, i.e.,

$$T''_g = T'_g\,(1 - n^*)\,, \tag{4}$$

we have [8]

$$\bar{T}_m(y) = T'_g \left\{1 - \exp\left[-\frac{\alpha_V S}{W_m}(1 - n^*)\, y\right]\right\}. \tag{5}$$

Substituting (4) and (5) into (2), we determine the gas temperature $T_g(y)$ on any horizon of the bed:

$$\bar{T}_g(y) = T'_g \left\{ 1 - n^* \exp\left[-\frac{\alpha_V S}{W_m} (1 - n^*) \, y \right] \right\} . \tag{6}$$

In [8], it is noted that relation (4) holds in completed heat exchange not only for thermally thin pieces of material but for massive ones as well. In practice, the process of countercurrent heat exchange may be considered to be completed if $T''_m \geq 0.95 T'_g$.

When $n^* > 1$, even when the gases give up their entire heat (completed process) and are cooled down to the temperature of the arriving charge ($T''_g = T'_m$), this will not ensure its heating to the initial temperature of the gases (i.e., we will always have $T''_m < T'_g$). In this case, unlike the previous one, the temperature difference between the gas and the charge will increase with descent of the charge. This difference and hence the underheating of the material will be the larger, the larger the number n^*.

Mathematically, the process of countercurrent heat exchange for $n^* > 1$ is described by the same equations as the process $n^* < 1$. When $n^* = 1$ the law of change of the gas and material temperatures over the bed's height is described by parallel lines:

$$\bar{T}_m(y) = T'_m + (T''_g - T'_m) \frac{\alpha_V S}{W_m} y , \quad T_g(y) = T'_g + (T'_g - T''_m) \frac{\alpha_V S}{W_m} y . \tag{7}$$

We note that the simplicity of the formulas for completion of the process ($n^* < 1$) under the actual conditions of their employment produces certain errors. It has been established that not only does the value of the error depend on the degree $\eta = \bar{T}''_m / T'_g$ of completeness of the process but it also depends on the parameter n^*. If $n^* << 1$, the errors are negligibly small as a rule. However, when $\eta^* < 0.95$ and $0.9 \leq n^* < 1$ the formulas considered yield a significant error.

To improve the accuracy it was proposed that the "apparent" heat capacity of the charge be employed in the calculations by replacement of the coefficient α_V by the total coefficient of heat transfer [8]

$$K_V = \alpha_V \left(1 + \frac{\text{Bi}}{5} \right), \tag{8}$$

allowing for the internal thermal resistance of the pieces ($R/\lambda \neq 0$). Owing to this technique, it became possible to artificially extend the procedure developed to the case of actual pieces possessing thermal inertia; however the accuracy of this approximate procedure amounts to $\pm$10–12%.

Works have also appeared in which the thermal massiveness of bodies is allowed for by solution of the boundary-value heat-conduction problem. The shape of the pieces of material is taken to be planar in some investigations [9] and spherical in others [1]; a generalized heat-conduction equation is employed in [3–5, 10–12].

Most of the existing works of this direction are devoted to filtration heat exchange in a dense bed; however in some works [2, 3, 9–11, 13], consideration is given to the countercurrent heat exchange of a metal in continuous furnaces.

In fact, [9, 13] are the first works in which countercurrent heat exchange is modeled by the boundary-value heat-conduction problem in a nonlinear formulation (nonlinearity in the Stefan–Boltzmann boundary condition). As far as the temperature dependence of thermophysical characteristics is concerned, it has been allowed for completely in none of the analytical works on countercurrent heat exchange up to the present time, although attempts have been made to take into account only the influence of the temperature on the heat capacity of a material [1, 8, 12, 13].

It is noteworthy that mathematical models of countercurrent heat exchange (CCHE) differ in the form of the taken condition of heat balance in interaction of the gas and the metal. In [14], this condition has been represented by the expression

$$T''_g - T_g(t) = [\bar{T}_m(t) - T'_m] \, n^* , \tag{9}$$

where the volume-mean temperature of the material is

$$\overline{T}_{\mathrm{m}}(t) = (1 + m) \int_0^R T(x, t)\, x^{1+m}\, dx\,. \tag{10}$$

A more accurate engineering procedure of calculation of the CCHE of spherically shaped massive bodies has been proposed in [8]. In this procedure, allowance is made for three terms of the Fourier solution, each of which contains five coefficients represented by the tables as functions of the Bi number (0.02–∞) and n^* (0.1–10). An advantage of this procedure is the high accuracy in determination of the gas and material temperatures. Among the disadvantages is the large volume of computations; moreover, the inverse (time) problem is difficult to solve. Furthermore, it considers the bulk mass of only spherically shaped pieces, whereas bodies of another geometry (plane, cylindrical) can also be found under the conditions of countercurrent heat exchange.

However, we can construct the procedure of calculation of heat-exchange processes in another way: from the onset, we can solve the mathematical problem formulated by any reliable approximate method.

The advantages of such an approach are as follows:

(1) the approximate method often enables one to allow for a larger number of parameters than the exact method;

(2) on the one hand, the final form of the approximate solution is much simpler than that of the exact one, on the other, it is more exact than the first term of the series appearing in the exact solution;

(3) the approximate methods enable one to solve a number of applied nonlinear problems whose exact solutions are absent.

All the above-mentioned solutions of the problems of countercurrent heat exchange have been considered in a linear formulation. This is attributed to the difficulties arising in nonlinear mathematical modeling.

Taking into account that all the features of nonlinear mathematical modeling for concurrent processes can be allowed for with the method of equivalent sources [15–17], we could concede that this method would also produce the same positive results in the problems of countercurrent heat exchange. Subsequent investigations [4, 5, 10, 11] confirmed these assumptions completely.

Convective Countercurrent Heat Exchange. Let us consider the corresponding linear boundary-value problem of heating of a thermally massive body of a base shape in a bounded gas volume.

A mathematical model will be taken in the formulation [2]

$$\frac{1}{\rho^m}\frac{\partial}{\partial \rho}\left(\rho^m \frac{\partial \theta_{\mathrm{m}}}{\partial \rho}\right) = \frac{\partial \theta_{\mathrm{m}}}{\partial \tau}\,, \tag{11}$$

$$\left.\frac{\partial \theta_{\mathrm{m}}}{\partial \rho}\right|_{\rho=1} = \mathrm{Bi}\,[\theta_{\mathrm{g}}(\tau) - \theta_{\mathrm{m,s}}(\tau)]\,, \quad \left.\frac{\partial \theta_{\mathrm{m}}}{\partial \rho}\right|_{\rho=0} = 0\,; \tag{12}$$

$$\frac{\partial \theta_{\mathrm{g}}}{\partial \tau} = \mathrm{Bi}\,[\theta_{\mathrm{g}}(\tau) - \theta_{\mathrm{m,s}}(\tau)]\, n_m^*\,, \tag{13}$$

$$\theta_{\mathrm{m}}(\rho, 0) = \theta_{\mathrm{m}}' = 0\,, \quad \theta_{\mathrm{g}}(0) = \theta_{\mathrm{g}}'' = 1\,, \tag{14}$$

where

$$\theta_{\mathrm{m}}(\rho, \tau) = \frac{T_{\mathrm{m}}(\rho, \tau) - T_{\mathrm{m}}'}{T_{\mathrm{g}}'' - T_{\mathrm{m}}'}\,; \quad \theta_{\mathrm{m,s}}(\tau) = \theta_{\mathrm{m}}(1, \tau)\,; \quad \theta_{\mathrm{g}}(\tau) = \frac{T_{\mathrm{g}}(\tau) - T_{\mathrm{m}}'}{T_{\mathrm{g}}'' - T_{\mathrm{m}}'}\,; \quad n_m^* = (1 + m)\, n^* = (1 + m)\frac{V_{\mathrm{m}} C_{\mathrm{m}}}{V_{\mathrm{g}} C_{\mathrm{g}}}\,;$$

$$\rho = \frac{r}{R}\,; \quad \tau = \frac{at}{R^2}\,; \quad \mathrm{Bi} = \frac{\alpha R}{\lambda}\,; \quad V_{\mathrm{g}} = \frac{R S_{\mathrm{h}}}{1 + m}\,. \tag{15}$$

The problem formulated is solved with the method of equivalent sources within the framework of the thermal-layer model [10, 18]. The essence of this approximate method lies in combining the method of successive approximations and the integral methods. The scheme of its use with the example of a one-dimensional heat-conduction equation is as follows. We write the $(n+1)$th approximation

$$c(\theta_n)\frac{\partial \theta_n}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(\theta_{n+1})\frac{\partial \theta_{n+1}}{\partial x}\right] + f_{n+1}(\tau)\,. \tag{16}$$

Integrating the initial heat-conduction equation and Eq. (16) for x between the limits $[0, l]$ and carrying out computation, we obtain

$$f_{n+1}(\tau) = \frac{1}{l}\int\limits_0^l \left\{c(\theta_n)\frac{\partial \theta_n}{\partial \tau} - c(\theta_{n+1})\frac{\partial \theta_{n+1}}{\partial \tau}\right\} dx\,. \tag{17}$$

Expression (17) is an integral condition of determination of the unknown function $f_{n+1}(\tau)$. Thus, if the nth approximation of the initial boundary-value problem is known, the $(n+1)$th approximation of $\theta_{n+1}(x, \tau)$ is determined by Eq. (16), the boundary conditions, and integral condition (17).

If the size of a body is bounded, the inertial step of heating is completed, as a certain time τ_0 passes $(0 \le \tau \le \tau_0)$, and an ordered step of heating throughout the cross section begins $(\tau > \tau_0)$. In the inertial step, the solution in the adopted coordinate system $(\rho = 1 - \xi)$ has the form [10]

$$\theta_{m1}(\rho, \tau) = \frac{\theta_{g1}\mathrm{Bi}}{[2 + \mathrm{Bi}\, l(\tau)]\, l(\tau)}[\rho - \beta(\tau)]^2\,, \quad \beta(\tau) \le \rho \le 1\,, \tag{18}$$

where

$$l(\tau) = \sqrt{6(1+m)k\tau}\,; \quad k = \frac{3 + 2\mathrm{Bi}}{3 + \mathrm{Bi}}\,. \tag{19}$$

To determine the temperature function $\theta_{g1}(\tau)$ we substitute the solution (18) into the heat-balance condition (13). After the corresponding transformations with the use of formulas (19) we arrive at a differential equation whose solution, with account for initial condition (14), yields

$$\theta_{g1}(\tau) = \exp\left\{M\left[l(\tau) - \frac{2}{\mathrm{Bi}}\ln\left(1 + \frac{\mathrm{Bi}\, l(\tau)}{2}\right)\right]\right\}, \tag{20}$$

where

$$M = \frac{2n^*}{3k}\,. \tag{21}$$

At $\tau = \tau_0 = (3 + \mathrm{Bi})/[6(1+m)(3 + 2\mathrm{Bi})]$, when the inertial step is completed, we have $l(\tau_0) = 1$ and the temperatures of the body and the gas are determined by the expressions

$$\theta_{m1}(\rho, \tau_0) = \theta_g(\rho, \tau_0)\,\frac{\mathrm{Bi}\,\rho^2}{2 + \mathrm{Bi}}\,, \tag{22}$$

$$\theta_{g1}(\tau_0) = \exp\left\{M\left[1 - \frac{2}{\mathrm{Bi}}\ln\left(1 + \frac{\mathrm{Bi}}{2}\right)\right]\right\}, \tag{23}$$

In the ordered step of heating $(\tau \ge \tau_0)$, the resolvent has the form

$$\frac{1}{\rho^m}\frac{\partial}{\partial\rho}\left(\rho^m\frac{\partial\theta_{m2}}{\partial\rho}\right)+f_2(\tau)=0\,, \tag{24}$$

$$f_2(\tau)=-(1+m)\frac{d}{d\tau}\int_0^l\theta_m(\rho,\tau)\,\rho^m d\rho\,. \tag{25}$$

Integrating Eq. (24) doubly for ρ and employing boundary condition (12), we obtain

$$\theta_{m2}(\rho,\tau)=\theta_{m,c2}(\tau)+\frac{\mathrm{Bi}\,[\theta_{g2}(\tau)-\theta_{m,c2}(\tau)]\,\rho^2}{2+\mathrm{Bi}}\,, \tag{26}$$

$$f_2(\tau)=-2\,(1+m)\,\mathrm{Bi}\,\frac{\theta_{g2}(\tau)-\theta_{m,c2}(\tau)}{2+\mathrm{Bi}}\,. \tag{27}$$

After substitution of the functions (26) and (27) into condition (25), we arrive at the differential equation

$$\frac{2+\mathrm{Bi}}{(1+m)\,\mathrm{Bi}}\frac{d\theta_{m,c2}}{d\tau}+\frac{1}{3+m}\left[\frac{d\theta_{g2}}{d\tau}-\frac{d\theta_{m,c2}}{d\tau}\right]=2\,[\theta_{g2}(\tau)-\theta_{m,c2}(\tau)]\,. \tag{28}$$

Setting $\rho = 1$ in the solution of (26), we find

$$\theta_{m,s2}(\tau)=\theta_{m,c2}(\tau)+\frac{\mathrm{Bi}}{2+\mathrm{Bi}}\,[\theta_{g2}(\tau)-\theta_{m,c2}(\tau)]\,, \tag{29}$$

after which we can write condition (13) as follows:

$$\frac{d\theta_{g2}}{d\tau}=\frac{2n_m\mathrm{Bi}}{2+\mathrm{Bi}}\,[\theta_{g2}(\tau)-\theta_{m,c2}(\tau)]\,. \tag{30}$$

Let us reduce expression (28) to the differential equation with separable variables

$$\frac{d\,[\theta_{g2}(\tau)-\theta_{m,c2}(\tau)]}{\theta_{g2}(\tau)-\theta_{m,c2}(\tau)}=-\frac{(1+m)\,\mathrm{Bi}}{1+\dfrac{\mathrm{Bi}}{3+m}}(1-n^*)\,d\tau\,; \tag{31}$$

integration of this equation, with account for initial conditions (22) and (23), yields

$$\theta_{m,c2}(\tau)=\theta_{g2}(\tau)-\theta_{g1}(\tau_0)\,\Phi(\tau)\,, \tag{32}$$

where

$$\Phi(\tau)=\exp\,[-\mu\,(\tau-\tau_0)]\,;\quad \mu=\frac{(1+m)\,\mathrm{Bi}\,(1-n^*)}{1+\dfrac{\mathrm{Bi}}{3+m}}\,. \tag{33}$$

Substituting expression (32) into (29), we find

$$\theta_{m,s2}(\tau)=\theta_{g2}(\tau)-\frac{2\theta_{g1}(\tau_0)\,\Phi(\tau)}{2+\mathrm{Bi}}\,. \tag{34}$$

The function of the gas temperature $\theta_{g2}(\tau)$ is determined after the integration of (30) with account for expression (32):

$$\theta_{g2}(\tau) = \theta_{g1}(\tau_0)\left\{1 + D\,[1 - \Phi(\tau)]\right\}, \qquad (35)$$

where

$$D = \frac{2n\left(1 + \dfrac{\mathrm{Bi}}{3 + m}\right)}{(2 + \mathrm{Bi})\,(1 - n^*)}. \qquad (36)$$

Now the solution of (26) takes the final form

$$\theta_{m2}(\rho, \tau) = \theta_{g1}(\tau_0)\left\{1 + D - \left[1 + D - \frac{\mathrm{Bi}}{1 + \mathrm{Bi}}\,\rho^2\right]\Phi(\tau)\right\}. \qquad (37)$$

Setting $\rho = 1$ and $\rho = 0$, we obtain the functions of the temperatures of the surface and center of the body:

$$\theta_{m,s2}(\tau) = \theta_{g1}(\tau_0)\left[1 + D - \left(\frac{2}{2 + \mathrm{Bi}} + D\right)\Phi(\tau)\right], \qquad (38)$$

$$\theta_{m,c2}(\tau) = \theta_{g1}(\tau_0)\,(1 + D)\,[1 - \Phi(\tau)]. \qquad (39)$$

The mass-mean temperature of the body is determined by the expression

$$\tilde{\theta}_m(\tau) = (1 + m)\int_0^l \theta_{m2}(\rho, \tau)\,\rho^m d\rho. \qquad (40)$$

After substitution of the function (37) and integration, we have

$$\tilde{\theta}_m(\tau) = \theta_{g1}(\tau_0)\left\{1 + D - \left[1 + D - \frac{(1 + m)\,\mathrm{Bi}}{(3 + m)\,(2 + \mathrm{Bi})}\right]\Phi(\tau)\right\}. \qquad (41)$$

The quantity n^* determines the character of the process. In the case of heating for $n^* < 1$, the process tends to completion with increase in the time t; the temperature functions of $\theta_{m,s}$, $\theta_{m,c}$, and θ_g converge and attain their common limiting value $\theta_m(\infty) = \theta_{g1}(\tau_0)(1 + D)$ at $\tau \to \infty$. If $n^* > 1$, the process is divergent: the longer the duration of heating, the larger the difference attained by the values of $\theta_{m,s}$, $\theta_{m,c}$, and θ_g. When $n^* = 1$, the approximate solution (35) and (37) obtained here has an indeterminacy of the form 0:0 whose evaluation leads to the following functions:

$$\begin{aligned} \theta_{g2}(\tau) &= \theta_{g1}(\tau_0)\left[2\,(1 + m)\,(\tau - \tau_0) + \frac{2 + \mathrm{Bi}}{\mathrm{Bi}}\right]\frac{\mathrm{Bi}}{2 + \mathrm{Bi}}, \\ \theta_{m2}(\rho, \tau) &= \theta_{g1}(\tau_0)\left[2\,(1 + m)\,(\tau - \tau_0) + \rho^2\right]\frac{\mathrm{Bi}}{2 + \mathrm{Bi}}. \end{aligned} \qquad (42)$$

As the investigations in [10] have shown, the exactness of the approximate solution obtained by the method of equivalent sources is quite sufficient for engineering purposes.

CONCLUSIONS

The analytical solution proposed here is quite acceptable for computation of the duration of heating of a metal in furnaces with countercurrent heat exchange, calculation of the temperature field over the cross section of the heated body at any instant of heating time, and determination of the dynamics of the temperature of the heating medium as a function of the consumption of fuel and the capacity of the furnace. This solution can be employed in cases where the linear mathematical model of countercurrent convective heating fits the actual process quite adequately. In cases where radiative heat exchange prevails and neglect of the temperature dependence of thermophysical characteristics is unacceptable, one must solve nonlinear boundary-value problems of countercurrent heat exchange to improve the accuracy.

NOTATION

$a = \lambda / c\gamma$, thermal diffusivity, m^2/sec; Bi = $\alpha R / \lambda$, Biot number, rel. units; C, heat capacity per unit volume, J/(m^3·K); c, specific heat, J/(kg^3·K); D, integration constant, rel. units; G, mass flow rate, kg/sec; H, bed height, m; K_V, total coefficient of heat transfer, J/(m^2·K); k, simplifying parameter, rel. units; l, dimensionless thickness of the thermal layer, rel. units; m, combining parameter of shape of a body (m = 0, plate; m = 1, cylinder; m = 2, sphere), rel. units; n^*, ratio of the water numbers of the material and the gas, rel. units; n_m^*, refined value of the parameter n^*, allowing for the parameter of shape of a body, rel. units; R, half the thickness of a plate, radius of a cylinder or a sphere, m; r, absolute coordinate reckoned from the center of the body's cross section, m; S, area, m^2; S_h, area of heating of a body, m^2; T, absolute temperature, K; $\bar{T}$, volume-mean temperature, K; t, time, sec; $V = RS_h(1 + m)$, volumetric rate of flow, m^3/sec; $W = Gc = CV$, heat capacity or water equivalent, J/K; w, rate, m/sec; x, coordinate, m; y, vertical coordinate, m; M, simplifying parameter, rel. units; Π, porosity, rel. units; Φ, temperature function in the Kirchhoff substitution, rel. units; α_V, volume heat-transfer coefficient, J/(m^3·K); γ, density, kg/m^3; $\eta = T''_m / T'_g$, degree of completeness of the heat-exchange process, rel. units; λ, thermal conductivity, W/(m·K); μ, root of the equation, rel. units; θ, relative excess temperature, rel. units; $\tilde{\theta}_m$, relative mean-mass temperature of a body, rel. units; τ, dimensionless time, rel. units; $\xi = 1 - \rho$; ρ, dimensionless coordinate, rel. units. Subscripts and superscripts: g, gas; m, material, solid body; h, heating; s, surface of a body; c, center of a body; n, order; V, volume; 0, initial (i = 1, 2), No. of heating step (inertial i = 1 or ordered i = 2); $'$ and $''$, values at entry and exit.

REFERENCES

1. M. A. Glinkov and G. M. Glinkov, *The General Theory of Furnaces* [in Russian], Metallurgiya, Moscow (1978).
2. É. M. Gol'dfarb, *Thermal Engineering of Metallurgical Processes* [in Russian], Metallurgiya, Moscow (1967).
3. V. I. Gubinskii and Zhun-U. Lu, *The Theory of Flame-Contact Furnaces* [in Russian], Mashinostroenie, Moscow (1995).
4. Yu. S. Postol'nik, V. I. Timoshpol'skii, O. A. Chernyi, D. N. Andrianov, V. I. Shcherbakov, and P. É. Ratnikov, Thermal and thermomechanical calculation of heating of massive ingots and billets in a countercurrent mode, *Lit'e Metallurg.*, No. 3, 165–170 (2003).
5. Yu. S. Postol'nik, V. I. Timoshpol'skii, O. A. Chernyi, D. N. Andrianov, and V. I. Shcherbakov, Solution of the problem of the process of heating of prismatic ingots in countercurrent, *Lit'e Metallurg.*, No. 2, 90–97 (2003).
6. B. I. Kitaev, *Heat Transfer in Shaft Furnaces* [in Russian], Metallurgizdat, Sverdlovsk (1945).
7. B. I. Kitaev, *Heat Transfer in Blast Furnaces* [in Russian], Metallurgizdat, Sverdlovsk–Moscow (1949).
8. B. I. Kitaev, Yu. G. Yaroshenko, and E. L. Sukhanov, *Thermal Engineering of Blasting* [in Russian], Metallurgizdat, Sverdlovsk (1978).
9. A. V. Kavaderov and V. N. Kalugin, Regularities of heating of a massive body by radiation in countercurrent flow, in: *Metal Heating and Operation of Heating Furnaces* [in Russian], Coll. of Sci. Papers, No. 6, Metallurgizdat, Sverdlovsk (1960), pp. 59–70.
10. Yu. S. Postol'nik, Calculation of temperatures and time of heating of massive bodies in countercurrent heat exchange, *Izv. Vyssh. Uchebn. Zaved., Chern. Metallurg.*, No. 6, 84–86 (1990).
11. Yu. S. Postol'nik and A. P. Ogurtsov, Comparative analysis of two mathematical models of heat transfer of massive bodies in countercurrent flow, *Matematichne Modelyuvannya*, No. 1 (4), 29–30 (2000).

12. G. D. Rabinovich, *Theory of Thermal Calculation of Recuperative Heat Exchangers* [in Russian], Izd. AN BSSR, Minsk (1963).
13. A. V. Kavaderov, *Thermal Operation of Flame-Contact Furnaces* [in Russian], Metallurgizdat, Sverdlovsk (1956).
14. V. N. Timofeev, Heat transfer in a bed of lumpy materials, in: *Regenerative Heat Transfer* [in Russian], Coll. of Sci. Papers, No. 8, Metallurgizdat, Sverdlovsk (1962), pp. 15–24.
15. Yu. S. Postol'nik, *Approximate Methods of Investigation in Thermomechanics* [in Russian], Vishcha Shkola, Kiev–Donetsk (1984).
16. Yu. S. Postol'nik, V. I. Timoshpol'skii, O. A. Chernyi, and D. N. Andrianov, Temperature stresses in prismatic bodies in radiative-convective heat transfer, *Lit'e Metallurg.*, No. 2, 98–104 (2003).
17. Yu. S. Postol'nik, V. I. Timoshpol'skii, O. A. Chernyi, and D. N. Andrianov, Comparative analysis of the thermally stressed state of a cylinder and a prism, *Lit'e Metallurg.*, No. 4, 42–45 (2003).
18. Yu. S. Postol'nik, V. I. Timoshpol'skii, and D. N. Andrianov, Calculation of the temperature fields of solid bodies of basic geometry under arbitrary boundary conditions, *Inzh.-Fiz. Zh.*, **77**, No. 2, 3–12 (2004).